Journal of Public Economics 15 (1981) 59–85. © North-Holland Publishing Company

LAND RENTS, OPTIMAL TAXATION AND LOCAL FISCAL INDEPENDENCE IN AN ECONOMY WITH LOCAL PUBLIC GOODS

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Received September 1978, revised version received April 1980

A general equilibrium model of an economy with cities, farms and free migration of population is constructed. The cities produce internationally traded goods via production functions subject to economies of scale. They also produce housing and a local public good. Two areas are defined to be disjoint if households performing an economic activity in one area are not operating in the other. An area is exclusive if it is disjoint to its complement. The economic surplus of an area is then defined to be the value of the area's net export of goods and resources. Local efficiency of an area is defined to be a state in which its economic surplus attains its maximum value. This state is proved to be a necessary condition for Pareto optimality of the economy. It is then proved that beside Piguvian corrective taxes the only taxes necessary and sufficient to finance local government activities efficiently, are taxes on land rents. Furthermore, if jurisdiction of a local government is over an exclusive area no intervention of central government is necessary, and local authorities can be fully autonomous. If the economy can be divided into pairwise disjointed exclusive areas, those areas are optimal jurisdictions in the sense that efficiency in the economy can be achieved with local authorities only.

1. Introduction

In this paper a general equilibrium model of an economy is constructed. In this model the existences of both a Tiebout (1956) type of local public goods and economies of scale in the production of goods that can be traded internationally lead to formation of cities. The ability of households to migrate freely between urban (cities) and rural areas ensures equal utility level to households with identical tasks, skills, and property, everywhere in the economy. Each city specializes in the production of a single tradable good and in providing its residents with housing and local public goods. City managers, whose aim is to achieve local efficiency by maximizing total city surplus, bring about a decentralized efficient equilibrium.

In the past decade the provision of local public goods by local governments has been the subject of quite a number of studies, for example those by Rothenberg (1970), Buchanan and Goetz (1972), Berglas (1976), Flatters, Henderson and Miezkovski (1974), Helpman and Pines (1977), Stiglitz (1977) and more.

*Partial financial support from the Center of Economic Research at the Desert Research Institute, Ben Gurion University is gratefully acknowledged.

These studies conclude that federal fiscal intervention is needed to achieve efficiency. The results of this paper indicate that there are two major reasons for this outcome. One is that some of these authors proposed to use local head taxes or local excise taxes. In this paper we show that there is only one tax that is allocative neutral — the tax on land rents. Any other tax is shifted to land rents and carries a dead weight loss with it. Once taxes on land are used, efficiency in the economy can be achieved by only local authorities. We thus return to the Old Ricardian[1] claim. Another problem which faced the above researchers was that local resources were considered insufficient to finance the activities of local government and sometimes transfers of income between cities were needed. Indeed, several economists beginning with Lind (1973), then followed by Flatters, Henderson and Mierkovski (1974), Helpman and Pines (1977), Hochman (1978), Arnott (1980) and especially Stiglitz (1977) in his outstanding contribution found out that land rents in the city equal government expenditure on local public goods. Stiglitz has been the only one, after Ricardo and Henry George, to have proposed utilization of land rents to finance the provision of local public goods. He may not have realized that this is the only way to do it efficiently. Yet even in his case, federal intervention is sometimes required to aid in the distribution of income between cities. A problem of a similar nature, namely the redistribution of (Piguevian) pollution tax proceeds between cities, faced Henderson (1977) and Berglas (1977) in their respective papers on pollution. The results in Hochman and Ofek (1979) show that if zoning regulations are used instead of Piguevian taxes, tax proceeds are then left to land owners as additional rents. It is shown here that distribution of land rights between households, *independently* of city of residency, can achieve the desirable income distribution between cities and that the intervention of central government is no longer necessary. In the literature we distinguish two types of local public goods. One type, which we may term a congestible public good, is the one utilized by club theorists, led by Buchanan. The quantity of services provided to each user by the public good is reduced when the number of users increases. This characteristic leads to the division of the population into clubs, each having its own congested public good and club members. Berglas (1976) proved that with free entry of clubs, congestion tolls in each club will exactly suffice to finance the local public good needed, and efficiency is therefore attainable without any central government intervention. No rents whatsoever exist in this model.

The other type of local public good investigated in the literature is a public good which is available only to those households which residue in its proximity. It can be a pure, congestible or any other type of public good. The congestible public good is therefore only a private case of the more

[1]Ricardo maintained that total land rents are sufficient to finance government activities and can be completely taxed away from owners.

general, second type. This type of public good is also the type discussed by Stiglitz. In his model a predetermined number of sites (islands) exist in which households cluster to consume and produce one private good and one public good. This type of local public good is the one utilized in this paper as well.

In the model developed here the formation of cities is endogenous. However, it is not limited to the purpose of providing a public good, as is the case with clubs, but also to utilizing economies of scale in the production of two types of traded goods as well as housing. External effects such as pollution and congestion are also accounted for as well as commuting costs and the provision of housing to each household in each city. An analysis of the short run (number of sites cannot be changed) and the long run (number of sites optimally determined) are then carried out. The identification of demand for public goods has long been a major problem and recently much effort has been devoted to this field (see, for example, Brock, 1980). In this paper a readily available *post ante* mechanism for the identification of the desirability of local government actions is revealed. It is argued that if land rents in the city improved, due to government action, by more than the government expenditure on the project, then the demand price for the project is higher than its cost and the project was carried out justly. The opposite is also true.

The question of optimal city jurisdiction is also investigated here. We find, following Bradford and Oates (1979), that the problem is more a matter of efficiency than of income distribution and that optimal jurisdictions are those which enable residents of the city to perform all their production and consumption activities in the same city. A full explanation as to when such optimal jurisdictions exist and how they function is included in the text.

The plan of this paper is as follows. Section 2 describes the underlying assumptions. In section 3 the general equilibrium conditions in the economy are specified. In section 4 local efficiency is defined as well as conditions under which cities are efficient in an economy in a state of equilibrium. Section 5 specifies conditions under which the economy is a Pareto optimum. In section 6 the question of optimal jurisdiction of local governments is investigated. In section 7 a speculation is carried out about possible extensions of the model. Conclusions and a summary of results are presented in section 8.

In the Appendix an optimization problem is formulated, the solution of which characterizes efficient resource allocation in the economy.

2. The underlying assumptions

Let us consider an economy which produces four marketed goods for consumption. Two of these goods are traded internationally, and under the assumption of zero transportation costs of final products, their prices P_1 and P_2 are everywhere fixed and exogenously determined. The third good, an agricultural commodity, is traded only within the economy and its price P_A is therefore also fixed everywhere in the economy but is internally determined. The fourth good, housing, is not even traded domestically, but rather is consumed on its production site. Its price R will therefore vary from location to location within the economy.

The economy has two production factors: land and labor. The total quantity of the former is fixed at L. The population of laborers who are the only city-dwellers consist of N households, each of which contributes one member to the labor force. N then is the total amount of labor available to the economy and, like L, is assumed to be fixed. Labor force participants have an additional source of income, namely ownership of property which can consist only of land. We assume all laborers own identical shares of property $\rho L/N$, $0<\rho<1$, where ρ designates the share of land owned by the labor force. The ownership of land yields income of V per household of labor force participants. Laborers own only part of the total land property. The rest of the land $(1-\rho)L$, is owned by pure landowners which brings them total income Y_0. Those pure landowners by assumption do not live in the economy's boundaries. In accordance with the theories detailed by Chipman (1970), Kemp (1969, ch. 8), Dixit (1973), Henderson (1974), and Hochman (1977a, b) it is assumed here that the traded goods produced in the economy have economies of scale of the following nature.

(a) When the returns to scale of an individual firm are constant, i.e. when the output of the industry is constant, equiproportional changes in the firm's inputs give rise to the same proportional changes in the firm's output.

(b) All economies external to the firm are (i) internal to the industry of which the firm is a member; (ii) generated by 'output' rather than 'factor'; and (iii) 'neutral' in the sense that at a given ratio of factor rentals the optimal factor ratio is independent of the industry's output.

(c) Economies of scale are realized through agglomeration in a single location.

Mills (1967, 1972), Dixit (1973) and Henderson (1974) introduced the last assumption in its present form, which leads to the formation of cities, into the literature of urban economics.

From the above assumptions it follows that the production functions of industries producing internationally traded goods are given by

$$X_i = g^i(X_i)F^i(A_i, N_i), \qquad i=1,2, \tag{1}$$

where X_i is the quantity of traded good i produced by A_i land and N_i labor. $F^i(\cdot)$ is linear homogeneous. Let θ^i be defined as

$$\theta^i = \frac{X_i(\mathrm{d}g^i(X_i)/\mathrm{d}X_i)}{g^i(X_i)}, \tag{2}$$

where θ^i is thus the partial elasticity of the individual firm's output with respect to the industry's output,[2] the assumption being that

$$0<\theta^i<1 \tag{3}$$

stems from the previous assumption of economies of scale.

The agricultural good X_A has no economies of scale in its production, and its neoclassical production function is given by

$$X_A = N_A f^A(I_A), \tag{4}$$

where I_A is land per farmer used in production, and $f^A(I_A)$ fulfils Inada's conditions. N_A is the number of farmers in the economy.

Housing, h, is produced by land alone, but workers must also incur the daily costs of commuting from their residence to the production sites of the traded goods.

Accordingly, not only will the industries producing internationally traded goods tend to agglomerate into a single location in order to benefit from production scale economies therein, but labor will also gravitate toward this site in order to minimize commuting costs. Any increase in the labor employed at a particular location will increase marginal commuting costs.

As pointed out by Henderson, the assumptions detailed above lead to the formation of cities, each of which specializes in the production of one traded good and provides housing to its labor force. The industry is located in a 'Central Business District' (CBD), and the residential section is adjacent to it.

We shall further assume that in addition to housing and traded goods, residents also consume a local public good (LPG) produced in the CBD by land and labor.[3] Let q_i be the LPG produced in the CBD of city i, and let the production function for q_i be linear homogeneous. The transformation curve of q_i and X_i, the local public good and the internationally traded good, is expressed as follows:

$$X_i = g^i(X_i)F^i(q_i, A_i, N_i), \tag{5}$$

where F^i is linear homogeneous. For a detailed analysis of the properties of

[2]To see that let $x=g(X)f(s,n)$, which is a production function of a firm producing quantity x with s land and n labor in an industry which produces X. Then the partial elasticity of the firm's output with respect to the industries output is $(\partial \ln x/\partial \ln X)=(\partial \ln g(X)/\partial \ln X)=\theta$. This proves our point. Further details can be found in Kemp (1969, ch. 8).

[3]Another common type of local public good is produced in the residential zone. Examples include roads and other public transportation facilities. Their inclusion in the present model leads to unnecessary complications, but the main results of the present paper hold for this case as well (see Hochman, 1975, 1978).

this transformation curve as implied by the individual production function, see Hall (1973).

Note that q_i can either be a desirable public good such as recreation facilities, public entertainment and services, radio and television broadcasts, or an undesirable public good such as pollution produced by the traded-goods industry. In the first case the derivative of F^i with respect to q_i is negative, while in the second case it is positive.

In summary then the transformation curve $F^i(\cdot)$ is assumed to be concave and linear homogeneous. Moreover,

$$\text{(a)} \quad F^i_1 \begin{cases} >0, & \text{iff } q_i \text{ local public bad,} \\ <0, & \text{iff } q_i \text{ local public good;} \end{cases}$$

$$\text{(b)} \quad F^i_2, F^i_3 > 0. \tag{6}$$

We shall refer to the city as city i, $i=1,2$ if in this city traded good i is produced.

The population in this model is assumed to be homogeneous with respect to tastes, skills, and ownership of property. Justification of identical tastes can be found in Stigler and Becker (1977), while the assumption of similarities in skills and property is merely for simplification. We assume each household contributes a single member to the labor force, then N_i is the number of households (laborers) in city i, and N_A is the total number of households in the agricultural sector, then let m_i be the number of cities of type i:

$$\sum m_i N_i + N_A = N. \tag{7}$$

We shall also assume that cities are linear with a given width (of unity) and the CBD and residential zone on either side. The concept of linear cities has become quite common in the recent urban economics literature. The difference between the linear city and the classical circular city is that in the former the supply of land at any given distance from the origin is constant, whereas in the latter it increases with distance. The expository simplification achieved by assuming city linearity is overwhelming in our case, while the loss in generality is negligible.[4]

Let L_i be both the length and total area of city i. Furthermore, let A_i be the length and area of the CBD. Since m_i is the number of cities of type i and N_A is the total number of farmers who use h_A land for housing and I_A land in the production of the agricultural good, the land constraint of the

[4]The additional simplifying assumption that the CBD and residential area are located on either side is not really restrictive at all, since one may argue that we are modelling half the economy, the other half being entirely symmetrical.

economy is

$$\sum_{i=1}^{2} m_i L_i + N_A(I_A + h_A) \leqq L. \tag{8}$$

Note that since no economies of scale exist in agricultural production, no agglomeration takes place in this sector which is instead characterized by single household production-cum-consumption units. We also assume that farmers do not consume local public goods.

Let $U(h,c,Z)$ be the utility function of the household where h is the amount of housing consumed by the household, c the amount of services of LPG consumed by the household, and z designates a vector of the two traded goods and the agricultural goods. Thus, the public good is not directly consumed but is used by the household, together with other inputs, to produce the direct consumption good c. Thus, we utilize here the Lancaster–Becker approach in its weaker sense.

Since housing is produced by land only, h is measured in units of land. The LPG consumed by the household depends on the amount of public good produced in the CBD, the distance of the location of the household from the CBD, and on the amount of income used by the household to either obtain the LPG, in cases where the LPG is desirable (DLPG), or in avoiding its bad effect in cases where the LPG is undesirable (ULPG). Thus,

$$c = C\,(q, D, e, N) \qquad \partial U/\partial h, \partial U/\partial Zj > 0,$$

$$C\,(0, D, e, N) = 0, \qquad \partial U/\partial c \begin{cases} >0, & \text{iff DLPG,} \\ <0, & \text{iff ULPG,} \end{cases}$$

$$C_q = \partial C/\partial q > 0,$$

$$C_D = \partial C/\partial D < 0,$$

$$C_e = \partial C/\partial e \begin{cases} >0, & \text{iff DLPG,} \\ <0, & \text{iff ULPG,} \end{cases}$$

$$C_N = \partial C/\partial N \begin{cases} <0, & \text{iff DLPG,} \\ >0, & \text{iff ULPG,} \end{cases}$$

in which the new variable, e, is expenditure on availability or avoidance of LPG depending on whether the LPG is desirable or not, and D is a parameter measuring the distance from the center of the city.

In the case of a DLPG, e may represent either travel expenses to the CBD for consumption of public goods such as attending museums or theaters, or having access to CBD facilities, or the expenses entailed in receiving broadcasts — radios, televisions, and antennae. In the case of a ULPG such as pollution e may be expenditures by residents on airconditioners, or on more frequent vacations.

The introduction of N into C represents congestion. In the case of DLPG, it would mean that for a given investment e a family located at D gets less services from the public good if the city is larger. Congestion in the case of ULPG means that damages from pollution to the individual household increases with density of population. If we take smoke pollution as an example, since high density of population implies high density of housing, a city with a large N is densely built and the smoke gets trapped for a longer period of time between buildings and is therefore more harmful and its avoidance is less effective.

The other consumption goods are represented by the vector

$$Z^i = (Z_1^i, Z_2^i, Z_A^i), \tag{10a}$$

where Z_j^i is the quantity of good j consumed by a household in city i, and $j, i = 1, 2, A$.

The price vector

$$P_Z = (P_1, P_2, P_A) \tag{10b}$$

consists of the market prices of these goods (net of taxes or subsidies). Let $t(D)$ be the commuting costs from a residential location at distance D from the center of town, for simplicity's sake assuming away the problems of congestion in transportation (see Hochman, 1975, 1978, for a discussion thereof). The characteristics of $t(D)$ are as follows:

$$t(0)=0; \qquad \mathrm{d}t(D)/\mathrm{d}D>0; \qquad \mathrm{d}^2t(0)/\mathrm{d}D^2<0. \tag{11}$$

The fact that there is free migration of population and equal shares in land owned by the labor force participants implies equal utility levels for all households participating in the labor force:

$$\begin{aligned}
&\text{(a)} \quad U(h_i(D),\ c_i(D),\ Z^i(D)) = U_0, \quad \text{for } A_i \leqq D \leqq L_i, \qquad i=1,2,\\
&\text{(b)} \quad U(h_A, 0, Z^A) = U_0,\\
&\text{(c)} \quad [\partial U/\partial Z_A]_{Z_A=0} = \infty,\\
&\text{(d)} \quad [\partial U/\partial Z_1]_{Z_1=0} = \infty.
\end{aligned} \tag{12}$$

Conditions (12c) and (12d) are intended to ensure that the household will always consume a positive quantity of the agricultural good and the internationally traded good 1. We also assume that $U(\cdot)$ is quasiconcave.

3. Equilibrium conditions in the economy

Let π_i be the price of good i facing producers — $i=1,2$. $\pi_i - P_i$ is the subsidy paid for good i to the producers in city i. Let W_i be the wage rate received by workers in city i at the center of town, and V be the nonearned income of a worker. Let R designate land rents, where $R_i(D)$ designates the rent of land paid by users to landowners located at distance D from the center of city i. Let $\bar{R}_i$ be the total land rents in city i:

$$\bar{R}_i = \int_0^{L_i} R_i(D)\,\mathrm{d}D. \tag{13}$$

Let $k_i(D)$ be a tax levied on a unit of urban land at location D in city $i, i=1,2$. Total taxes on land in city i are

$$\bar{K}_i = \int_0^{L_i} k_i(D)\,\mathrm{d}D. \tag{14}$$

Note that landowners will hold on to their land only as long as $k_i(D) < R_i(D)$. Otherwise landowners will abandon their property and they, as well as the government, will not make any money. Thus, we assume rational governments, i.e.

$$k_i(D) \leqq R_i(D); \qquad 0 \leqq D \leqq L_i; \qquad i=1,2. \tag{15}$$

Let TR be the landowners' total net income from land, then

$$TR = \sum m_i(\bar{R}_i - \bar{K}_i) + \bar{R}_{\mathrm{A}} \geqq 0, \tag{16}$$

where $\bar{R}_{\mathrm{A}}$ equals total land rents in the agricultural sector. The nonearned income of a household is equal to the household's share of TR, i.e.

$$V = \rho TR/N. \tag{17}$$

The rest of the land rents go to pure landowners, thus

$$Y_0 = (1-\rho)TR. \tag{18}$$

Let $b_i(D)$, $i=1,2$, be a head tax levied on households living in city i at distance D from the center. Taxes on land to users are considered by the users to be part of the rent, and therefore it is equivalent to k_i. Taxes on wages by users of labor are also passed on to workers and thus equivalent to b_i. Although here we are not investigating relative income tax, the results we shall obtain can easily be extended to include this case as well.

Let σ_{ij}, $j=1,2,\mathrm{A}$, be the consumption goods prices of z_i facing consumers in city $i, i=1,2$. Therefore $\sigma_i=(\sigma_{i1},\sigma_{i2},\sigma_{i\mathrm{A}})$ is the price vector facing consumers in city i. The price P_{q_i} is the payment per unit of q_i paid by the local government to the industry for the production of q_i. The price P_{q_i} can be either negative (in the form of taxes) or positive (in the form of market price or subsidy).

3.1. Equilibrium in the residential ring of city i

The household's budget constraint is given by

$$V+W_i=R_i(D)h_i+\sigma_i Z+e_i+t(D)+b_i(D);\quad A_i\leqq D\leqq L_1. \tag{19}$$

Maximization of the household's utility function subject to (19) implies

$$\frac{R_i(D)}{U_h(i,D)}=\frac{\sigma_{ij}}{U_{Z_j}(i,D)},\qquad i=1,2;\quad j=1,2,\mathrm{A};\qquad A^i\leqq D\leqq L_i, \tag{20}$$

where $U_X(i,D)$ designates the value of the derivative of U with respect to variable X in city i at location D.

$$U_{Z_1}(i,D)/\sigma_{i1}=U_C\cdot C_e, \tag{21}$$

where $C_e=\partial C/\partial e$.

3.2. Equilibrium conditions in the CBD

Producers act to maximize the firm's, and therefore the total industry's, net gains in city i, given product and factor prices. Since $g(X_i)$ is exogenous to the individual firm we obtain as necessary conditions:

$$\pi_i g^i(X_i)F_\mathrm{A}=R_i(A_i). \tag{22}$$

Since F^i is linearly homogeneous, land rents everywhere in the CBD are the same. Hence,

$$R_i(D)=R_i(A_i),\qquad 0\leqq D\leqq A_i. \tag{23}$$

In the same way we also have

$$\pi_i g^i(X_i)F^i_N=W_i, \tag{24}$$

$$-\pi_i g^i(X_i)F^i_q=P_{q_i}. \tag{25}$$

Equality between the supply and demand for labor follows from the next equation:

$$\int_{A_i}^{L_i} \frac{1}{h_i(D)}\,\mathrm{d}D - N_i = 0. \tag{26}$$

The term $1/h_i(D)$ signifies the number of households at location D. Summing this over the residental strip yields the total number of households in the city. By assumption every household contributes one worker to the CBD; thus, the integral expression in (26) is also equal to N_i, the number of workers in the CBD.

3.3. Equilibrium conditions in the agricultural industry

$$R_A = P_A f_1^A(l_A), \tag{27}$$

where l_A is the land used by a farmer in his production of the agricultural good. Since W_A is the income of a farmer from the production process, we have

$$W_A = P_A f^A - R_A l_A. \tag{28}$$

The price P_A in our model is determined endogenously in such a way that the total agricultural good produced is equal to the total agricultural good consumed:

$$\sum m_i \left(\int_{A_i}^{L_i} (Z_A^i(D)/h_i(D))\,\mathrm{d}D \right) + N_A Z_A^A = X_A. \tag{29}$$

The agricultural land rent, R_A, is such that the land constraint in the entire economy [eq. (8)] holds.

3.4. Equilibrium conditions for a farmer's household

The household budget constraint in the agricultural sector is as follows:

$$V + W_A = R_A h_A + P_z \cdot Z^A. \tag{30}$$

Consumer's behavior is therefore characterized by

$$\frac{R_A}{U_h(A)} = \frac{P_i}{U_{Z_i}(A)}, \qquad i = 1, 2, A. \tag{31}$$

Recall that $C(A)=q_A=e_A=0$ by assumption. Let the jurisdiction of a local government coincide with L_i.Let b_i, π_i, σ_{ij} and k_i, $i=1,2$, $j=1,2,A$, be prices and taxes set by the local government. We consider taxes by urban authorities only. Each local government operates by assumption, under a balanced budget, then

$$\bar{K}_i+\int_{A_i}^{L_i}\frac{b_i(D)}{h_i(D)}\,\mathrm{d}D-P_{q_i}q_i+(P_i-\pi_i)X_i$$
$$+\int_{A_i}^{L_i}\left[\frac{1}{h_i(D)}\sum_j(\sigma_{ij}-P_j)Z_j\right]\mathrm{d}D=0;\qquad i=1,2. \tag{32}$$

For every set of exogenously determined prices, taxes, and subsidies, P_z, π_i, σ_i, b_i, P_{q_i}, and k_i, which satisfy eqs. (1)–(32), there is an equilibrium solution. In particular, for $\pi_j=\sigma_{ij}=P_j$, $j=1,2$, $b_i=P_{q_i}=k_i=0$, $i=1,2$.

The above equilibrium is the *laissez-faire* solution. In this case the DLPG q_i is not produced at all and the ULPG q_i is produced up to the level in which (25) is satisfied for $P_{q_i}=0$.

The assumptions specified in (12c) and (12d) imply that each household must consume some quantity of the agricultural good and traded good 1. Since the agricultural good can be produced only in the economy, some agricultural activity must take place. At least one of the goods 1 or 2 must be produced in the economy, otherwise, there would be no good 1 to consume. If it is not produced in the economy, it cannot be bought since international trade is impossible without the production of at least one of the traded goods.

If the economies of scale are not exhausted by the diseconomies of scale in the production of housing, only one city will exist in the economy, producing only one type of traded good. We will not investigate this case here, and assume that many cities of one or two kinds exist. It should be noted that in our model the land constraint is always effective, i.e. $R_A>0$ and no vacant land is left. This is so since no transportation costs between cities exist and since the agricultural production function is linearly homogeneous. Those two reasons imply that agricultiral production must be spread' uniformly over all of the nonurban land.

4. Efficient resource allocation in the cities

Definition 1. Consider an area of land in an economy in equilibrium (such as the one described in the previous section) in which activity of consumption and/or production is taking place. The surplus of the area is defined as the market value of all exports of goods and services of the area minus the market value of all imports.

The size of the area need not be predetermined as long as it is well defined. The surplus of an empty area is nil. Thus, the value of exports of an area in which a household is located is the sum of the earned and nonearned income minus commuting expenditures, and the value of the imports of the area is the sum of the value of all consumption goods imported into the area and the expenditure involved in the consumption of the public good.

Definition 2. We say that two areas are intersecting if there is at least one household which is active in both areas. The household may fulfil different roles in the two areas because it resides in one and works in another. Two areas that are not intersecting are said to be disjoint. Thus, the residential rings of a city intersect the CBD of the same city, while two different cities are disjoint.

Definition 3. An area which is disjoint to its complement (the rest of the economy) is said to be exclusive. Thus, all cities and farms in our economy are exclusive.

Lemma 1. The following are necessary conditions for efficient allocation of resources in an economy in equilibrium.

(i) *In each area in the economy the net surplus attains its maximum value subject to the following constraints:*

(ii) *That real rewards be equal to identical mobile factors. In our model the only mobile factor is labor and equal real rewards is equal utility level.*

(iii) *Production and consumption technologies.*

(iv) *Balanced supply and demand of resources in the area.*

Proof. Let S designate the net surplus of the area. To prove necessity we shall assume S is not at its maximum value and then show that Pareto conditions for optimality are not fulfilled. Suppose the value of S is S_1, is not maximized. Then we can increase S to a higher value S^*, keeping the real rewards of factors constant (such as the utility level of the population), according to condition (ii). The additional surplus created this way, $S^* - S_1$, can now be used to make somebody in the economy better off without penalizing anyone else. Thus, if S is not at its maximum value, a Pareto condition for efficiency is not fulfilled — a contradiction. Therefore, for the Pareto conditions to be fulfilled we must have S at its maximum value. Q.E.D.

Definition 4. An area is said to be locally efficient if it fulfils the conditions of lemma 1. Let S_i be the net surplus of city i

$$S_i = P_i X_i + VN_i - \int_{A_i}^{L_i} \frac{\mathrm{d}D}{h_i(D)} \times [P_z \cdot z + e + t(D)] - R_A L_i, \tag{33}$$

where $P_i X_i$ is income from sales of export good i produced in the city and VN_i is total nonearned income of the city's population. The value V is exogenously given and so is R_A. $[(!/h(D))]$ is the number of households per unit of land at distance D, and the term in the brackets in the right-hand side (RHS) of eq. (33) is total expenditure of a household in terms of income (i.e. costs of imported goods consumed and direct expenditure of income). Note that expenditure on housing and on the public good are not included since both are completely produced and consumed in the city.

Proposition 1. The maximization of S_i subject to the equal utility constraint [eq. (12)], the production possibilities frontier in city i [eq. (5)], and the city population constraint [eq. (26)] is a necessary condition for a Pareto optimal allocation of resources in the economy.

Proof.[5] The proof follows directly from lemma 1.

The necessary conditions of maximization of S_i subject to (5), (12) and (26) are eqs. (19)–(25) with

$$\sigma_{ij} = P_j, \tag{34}$$

$$b_i = -\int_{A_i}^{L_i} \frac{P_1}{h_i(D)} \frac{U_c}{U_{Z_1}} C_N, \tag{35}$$

$$\pi_i = P_i/(1-\theta^i), \tag{36}$$

$$P_{q_i} = \int_{A_i}^{L_i} \frac{P_1}{h_i(D)} \frac{U_c}{U_{Z_1}} C_{q_i}. \tag{37}$$

Eq. (34) shows that no taxes or subsidies are to be levied on consumption goods. Eq. (35) gives us the congestion toll to be levied on a household in accordance with Pigou's theory. The term on the right-hand side of eq. (35) equals the damage caused to the total city population by a marginal increase in city size, the damage being caused by a reduction in the services of the public good, q_i, to each household due to congestion. When no congestion effects are present, i.e. $C_N = 0$, b_i should equal zero as well. Thus, any head taxes other than the optimal congestion tolls b_i^* is bound to carry excess burden.

Eq. (36) indicates that product prices facing the industry should be higher than market prices, i.e. the industry should be subsidized to internalize the

[5] In the appendix the necessary conditions for Pareto optimality in our economy are calculated. It can be seen that the necessary conditions for maximization of S_i, which follows in this section, coincide with some of the necessary conditions for efficiency in the rest of the economy, thus providing the necessity of maximizing S_i to achieve efficiency in a less general way.

external economies of scale, again in accordance with Pigou. When substituting from (37) for P_{q_i} in (25) we obtain Samuelson's well-known necessary condition for efficient allocation of public goods, i.e. the rate of product transformation between the public and the private good should be equal to the sum over all individuals of the household's marginal rates of substitution between the public and private goods. To achieve this relation the government has to pay the industry P_{q_i} per unit of q_i, where P_{q_i} is given by (25). Note that if q_i is a public bad (ULPG) then P_{q_i} can be interpreted as Pigou's corrective tax. Thus, Samuelson's and Pigou's well-known relations are actually the same, the first related to a desirable PG and the second to an undesirable one.

Of all the prices and taxes listed in eq. (32), which include all taxes and prices under the control of a local urban authority, the only ones not yet determined are taxes on land, k_i, and through them $\bar{K}_i$. The land taxes, k_i, are taxes which do not effect resource allocation. This characteristic of land rents was already noticed by Ricardo (1957) and can also be found in Mills (1972, ch. 3). Landowners do not decide upon the nature and intensity of the use made of their land. What they do is to choose the highest bidder for the land. It is the renter who makes the decision about how to use the land.

If taxes on land are independent of the particular use made of the land, the motivation and behavior of landowners are not going to change, since they will still have a higher net income from their land if they lease it to the highest bidder. Thus, we just showed that taxes on land are the *only* allocation neutral taxes available to local authorities. This is so since, as we showed earlier, head taxes are reserved for congestion tolls and changing them beyond the amount specified in (35) will lead to distortions in the allocation. The same applies to all other types of taxes, except taxes on land.

Corollary 1, now follows directly.

Corollary 1. A local government can achieve local efficiency in its jurisdiction if, except for corrective Pigouvian taxes and subsidies, the only source of financing for its activities (such as the provision of DLPG and internalizing externalities) is from taxes on land rents.

Substituting in (32) the optimal values of the prices and taxes, we obtain

$$\bar{K}_i = N_i \int_{A_i}^{L_i} \frac{P_1}{h_i(D)} \frac{U_c}{U_{Z_1}} C_N \,\mathrm{d}N + q_i \int_{A_i}^{L_i} \frac{P_1}{h_i(D)} \frac{U_c}{U_{Z_1}} C_{q_i} \,\mathrm{d}D + \frac{\theta^i}{1-\theta^i} X_i. \tag{38}$$

Since $\bar{K}_i$ is constrained by total land rents [see eq. (15)] it should be interesting to discover if and when sufficient funds cannot be obtained for the financing of the local government by only taxing land rents. The function F^i is linearly homogeneous, therefore Euler's law holds for it. Thus,

$$F^i(\cdot)=q_iF^i_q+A_iF^i_A+N_iF^i_N. \tag{39}$$

Multiply (39) by $\pi_i g^i(X_i)$ and obtain, after using (5),

$$\pi_iX_i=\pi_ig^iF^i_q\cdot q_i+\pi_ig^iF^i_AA_i+\pi_ig^iF^i_N\cdot N_i.$$

Substituting (22)–(25) into the above, we obtain:

$$-\pi_iX_i-P_{q_i}\cdot q_i+R_i(A_i)\cdot A_i+W_iN_i=0. \tag{40}$$

By adding (40) to the right-hand side of (33), we obtain:

$$S_i=(P_i-\pi_i)X_i-P_{q_i}\cdot q_i+R_i(A_i)\cdot A_i+(W_i+V)N_i$$
$$-\int_{A_i}^{L_i}\frac{\mathrm{d}D}{h_i(D)}[P_z\cdot z+e_i+t(D)]-R_AL_i.$$

Substituting the budget constraint (19) into the integral term in the above expression, we obtain:

$$S_i=(P_i-\pi_i)X_i-P_{q_i}\cdot q_i+\bar{R}_i-R_AL_i$$
$$+\int_{A_i}^{L_i}\frac{b_i(D)}{h_i(D)}\mathrm{d}D+\int_{A_i}^{L_i}\left[\frac{\mathrm{d}D}{h_i(D)}\sum_j(\sigma_{ij}-P_j)Z_j\right]. \tag{41}$$

Substituting (32) into (41), we obtain:

$$S_i+\bar{K}_i=\bar{R}_i-R_AL_i. \tag{42}$$

There are several important conclusions following from eqs. (41) and (42), but before we proceed with them let us first prove the following lemma.

Lemma 1a. Let S^* be the optimal value of S_i, where S is the surplus of an area. Then $S^*\geqq 0$.

Proof. Consider the solution to the equilibrium in which no activity whatsoever takes place in the area. The value of S is then nil. This case is a feasible solution and thus constitutes a lower bound of the maximum value of S. Hence, if S^* is the maximum value of S then we have

$$S^*\geqq 0. \qquad \text{Q.E.D.} \tag{43}$$

Actually, what we are saying here is that if S_i cannot exceed zero no city of type i will exist in the economy. A negative value of S in a city implies that the local government is not operating with a balanced budget, and this case is ruled out by assumption in our model. Thus, we have actually created a system in which S must be non-negative by assumption. It only remains to show that such a system yields an efficient solution. This will be proved in the next section. In the meantime, let us return to our analysis of eqs. (41) and (42).

Corollary 2. Any local government's tax, subsidy, or expenditure on local public goods is passed on to land rents. The welfare gains or losses of such actions are measured by the change in S_i, and are also passed to land rents.

The corollary is obvious in the case of taxes and subsidies on land. The arguments for other types of local taxes, subsidies, and local government expenditure follow directly from eq. (41). To see this more clearly, let us rewrite eq. (41) as follows:

$$\bar{R}_i - R_A L_i = S_i + (\pi_i - P_i)X_i + P_{q_i} q_i + \int_{A_i}^{L_i} \frac{b_i(D)}{h_i(D)}\,\mathrm{d}D + \int_{A_i}^{L_i} \left[\frac{\mathrm{d}D}{h_i(D)} \sum_j (P_j - \sigma_{ij}) Z_j\right]. \tag{44}$$

Thus, we see that any tax, or other local government expenditure on goods and services within its jurisdiction is immediately passed to land rents. This proves the first part of the corollary. Those actions also affect S_i since efficiency is achieved when S_i is maximized. Any government action which increases S_i is therefore a step taken in the right direction and any action which decreases S_i is a step taken in the wrong direction. The change in S_i can therefore be considered as a measure of the welfare of the action which caused it.

That land rents reflect some local government actions has been recently the outcome of a number of studies. Here the results are generalized to any equilibrium solution — efficient or not — and to any local government activity, be it taxation, subsidization, or an investment in a local public good. All previous works dealt only with government investments and only in efficient models in which S_i was equal to zero.[6] We also see that a local government action is transferred to urban land rental. By urban land rent we mean that part of the land rent which exceeds the alternative nonurban land rent, i.e. the agricultural land rent R_A.

[6]As we shall show later, S_i is equal to zero in the long run with free entry of cities. It can be shown that this is true also where no entry or exit of population is possible.

We come now to our next corollary which is a direct implication of eq. (42).

Corollary 3. A local government has an a posteriori *criterion with which to judge the efficiency of its actions. If after the government action total land rents increase more than the total expenditure involved in the action, then this action has increased the efficiency in the city, and is thus a positive step. If total land rents increased by less than the expenditure on the government's project, then this project has contributed to a decrease in efficiency.*

This corollary is a direct consequence of eq. (42). Since an increase in the value of S_i is the objective, it is clear from (42) that if the increase in $\bar{R}_i - R_A L_i$ exceeds the increase in $\bar{K}_i$, which measures the government expenditure on the project, then S_i must have increased as well. The opposite results when the increase in $\bar{R}_i - R_A L_i$ is exceeded by the increase in $\bar{K}_i$.

Corollary 4. A local government can provide its own financing to achieve local efficiency without any intervention from the federal government. No help is needed either to finance DLPG or to help redistribute tax proceeds from corrective Pigouvian taxes.

We now need only to prove that local efficiency leads to Pareto efficiency in the entire economy to show that fiscal federalism, advocated by studies on desirable and undesirable local public goods through the seventies, is not really needed. We shall return to this question when the efficiency in the entire economy is discussed in the next chapter.

5. Efficiency conditions in the economy

As was done in the case of cities, let us define the surplus function of a farm,

$$S_A = P_A f^A(l_A) + V - \sum_{i \in (1,2,A)} P_i Z_i^A - R_A(h_A + l_A), \tag{45}$$

where R_A is the equilibrium rent of agricultural land and P_A is the market price of the agricultural good. Necessary conditions for the maximization of S_A subject to (12) are given by eqs. (27) and (31). Since the prices are the equilibrium prices, (28) and (30) hold as well. Therefore the efficient and competitive allocations are identical. Furthermore, by substituting eqs. (27), (28), (30) and (31) into (45), we obtain

$$S_A = 0. \tag{46}$$

Let us define the production and consumption area in which the agriculture activity is taking place as a farm. The farm's land is thus equal to $l_A + h_A$. Farms, like cities, are exclusive. The above discussion can now be summed up in the following proposition.

Proposition 2. Farms in our economy are locally efficient.

Lemma 2. The union of two disjoint locally efficient areas in an economy in equilibrium is also locally efficient.

Proof. The surplus function of the union of the areas is the sum of the surpluses of the two areas. Hence, maximization of each of those areas' surpluses implies the maximization of the sum of their surpluses under the same constraints as each of the units.

Lemma 3. The union of any number of pairwise disjoint, exclusive, locally efficient areas in an economy in equilibrium is locally efficient.

The proof follows from lemma 2 by adding units successively.

Lemma 4. Suppose an economy is divided into pairwise disjoint, exclusive, locally efficient areas, then the whole economy is locally efficient.

The proof follows from lemma 3. Since the economy can be presented as a union of pairwise disjoint, exclusive, locally efficient areas.

Lemma 5. A locally efficient economy in which the total net surplus is completely distributed between agents in the economy is globally efficient (i.e. is Pareto optimal).

Proof. We will assume as a working hypothesis that the lemma is wrong and that the economy is not globally efficient and prove that the assumption leads to a contradiction. Since the economy is not globally efficient (but it is locally efficient), we can raise the utility level of at least one individual in it. Suppose that individual 1 is such an individual. Since individual 1's utility is now higher than it was before, we can reduce his income so that his utility will drop back to its previous level and in this way create an additional surplus of income.

This contradicts our assumption that all surplus is distributed between agents in the economy, since this newly created surplus is not distributed. Hence, the starting assumption that our economy is not globally efficient leads to a contradiction. This proves that the economy is Pareto optimal. Q.E.D.

Proposition 3. When all cities and all farms in our economy are locally efficient, then the whole economy is Pareto optimal.

Proof. The union of all farms and cities in our economy completely exhausts the economy. All farms and cities are pairwise, disjoint, and exclusive, since from lemma 4 the whole economy is locally efficient. From lemma 5 it follows that our economy is Pareto optimal. We come now to our major result.

Corollary 5.[7] *An economy with local public goods can operate efficiently with only local authorities and without any federal intervention.*

The proof follows directly from corollary 4 and proposition 3. These results contradict some previous studies in the field of local public goals made in the last decade. These studies found that efficiency cannot be achieved by local authorities by themselves, and that federal fiscal intervention is needed. The question is, what caused these scholars to reach this conclusion? One reason is that some of them suggested financing local government activities by levying local head taxes; which we proved to be inefficient but are known to be allocatively neutral when used by a federal government. This is the reason for the need for federal intervention to achieve efficiency. When the head tax is levied by the central government no excess burden is involved.

Income distribution is another factor which needed federal intervention. Several researchers noted that at times income has to be transferred from one type of city to another in order to ensure an equal utility level to identical households in different cities. To achieve that purpose federal intervention was needed again. This was true in both the case of ULPG and DLPG.

The solution to the problem of transfer of income between different types of cities listed in this model also deals with land rents. The income which needs to be redistributed is the total surplus of the economy which is the sum of the surplus of all the cities in the economy. In this study it is redistributed as land rents. From substituting (42) into (16) we obtain

$$TR=\sum_i m_i(S_i+L_iR_A)+N_A(I_A+h_A)R_A. \tag{47}$$

Part of this income is distributed to pure landowners [eq. (18)] and the rest is distributed equally between the rest of the population as nonearned income, V. In a system in which there are no land rents there is no way to distribute an equal share of this income to equal individuals in different cities except by resorting to a federal head subsidy (or tax). Thus, we see that

[7] A direct and less general proof to the same effect is pointed out in the appendix.

again the 'hidden hand' on the market system has a way of doing the (federal) government's job.[8] It should be noted that it need not be that everybody owns land in all cities. It is quite possible that in order to minimize property managing costs (which do not exist in our model) everybody owns land as close to his place of residence as possible. The only constraint is that identical individuals receive identical income from their respective properties.

In the long run there is free entry of cities. New cities of type i will continue to enter as long as $S_i > 0$, since positive S_i allow cities to attract resources by offering higher rewards than elsewhere.

A long-run equilibrium condition in our economy is therefore

$$S_i = 0. \tag{48}$$

In the long run, inefficient cities cannot survive. Then inefficiency means that S is not at its optimal value; hence, more efficient cities with higher S will be able to attract labor away from other inefficient cities. In the long run, since S is zero, urban land rents exactly match local government expenditure and so Ricardo's and George's theory that land rents are exactly sufficient to finance the supply of public goods exactly match our results with respect to LPG.

6. Optimal jurisdiction of local governments

Another important implication of our analysis so far concerns the question of optimal jurisdiction of local governments. First, we should point out again that local governments can affect income distribution only between owners of a city's land because of the mobility of households and other factors (if they exist) excluding land throughout the economy. Suppose, for example, that a local government improved living conditions of a certain population group in a certain location. The demand for housing in this area will increase and cause an increase of rents and with it the income of landlords in this location, but the utility level of the particular population group remains unchanged. Note that to keep utilities fixed at an economy-wide level, only a small fraction of the population has to be actually mobile. A slight increase in the density of the population, or even only in the number of those searching for housing, is sufficient to drive housing rents up and decrease the utility level of tenants. In the same way, a few additional empty flats will diminish rents and the prices of housing and increase the utility level of

[8]This is true, of course, only if a local government can be considered to be an economic entity operating in competitive markets. Exactly such an approach is detailed in a recent mimeograph by the author: 'A theory on the behavior of municipal governments'.

tenants. Since a fraction of the population, especially in the big cities, is always moving, there is no cost of adjustment involved in keeping the utility level of a population constant. Thus, the concern of some writers about the distributive effects of some local governments with respect to their underprivileged population groups is premature. No such effect exists. The only transfers of income taking place are between landowners.

Let us now turn to the question of what is the optimal jurisdiction of a local government. We already know, from corollary 1, that the only financial resources available to a local government, besides net Pigouvian taxes which are allocatively neutral, are taxes on land, k_i. From eq. (42) we learned that those taxes are exactly equal to total land rents minus the alternative value of land and net city surplus. If the area of residency of the households who work in a city and consume city public goods is not totally contained in the city's jurisdiction, the city will have to resort to either one or both of the following possibilities, in order to finance its activities:

(a) tax landowners in the city's jurisdiction in a way that net income from their land will drop below the level of alternative agricultural land rent; and/or

(b) use excise and head taxes in addition to land taxes, such as income tax, sales tax, etc.

If the municipality were to choose alternative (a) it will transfer income from property owners inside its jurisdiction to property owners outside its jurisdiction. Clearly, if that happens we can say that the suburban landowners are exploiting central city landowners. If the local government were to choose alternative (b) it will introduce inefficiency into the system. Quite often, when the problem of insufficient jurisdiction of a central city and of a population, a high percentage of whom live outside jurisdiction, is of major proportions, the central city government has to use both alternatives. This usually happens in huge metropolitan centers such as New York, Philadelphia, or Chicago, in which suburbanites constitute over 50 percent of the population in the greater metropolitan area.

Direct political and other pressure from local landowners usually causes the municipality to shift increasingly from alternative (a) to (b), thus increasing inefficiency in favor of a more desirable income distribution between local and nonlocal property owners. This concept was indeed pointed out by Bradford and Oates (1979), namely that the problem of efficiency is the major one involved in the question of optimal jurisdiction. The optimal solution to this problem, which follows from our model here, coincides with that of Bradford and Oates, namely that the jurisdiction of the central city should include all places of residency of people working and/or consuming DLPG in the city, and all places of work and consumption of DLPG of city residents, thereby creating in the author's words 'a united system encompassing (at least) the entire metropolitan area'. The above discussion is summarized in corollary 6.

Corollary 6. Optimal jurisdictions of cities are those which allow acts of production and consumption of households to be performed in the same jurisdiction.

In our terminology we can say that an area is optimal, as that of the jurisdiction of a local government, if it is exclusive. If nontrivial division of the economy into exclusive areas is not possible, efficiency cannot be achieved without central government intervention. It is still possible, however, to define jurisdictions in such a way that will minimize spillovers, and then, in addition, to let central government treat local governments as separate units and subsidize or tax them to achieve efficiency. This type of problem and solution is discussed in Hochman and Ofek (1979) with respect to pollution spillovers. A more comprehensive treatment of this subject requires a special study.

7. Possible extensions of the model

The discussion of the possible extensions of the model is neither rigorous nor detailed in its proofs, yets its strong intuitive appeal justifies its inclusion in this discussion. The model can be extended to the case in which the nonearned income is distributed unequally between the labor force by having several population groups with equal tastes and skills but different shares of the property. Let ρ_j be the share of population group j in the land property

$$\rho_j \geqq 0, \qquad j=1,2,\dots,J,\ J\geqq 3 \text{ and } \sum_j \rho_j = 1.$$

The only additional result is that these population groups may choose to live in different cities; if they should choose to live in the same city, then they will most probably choose different locations in the city. Otherwise, the result previously discussed should not be affected. In the same way the discussion can also be extended to include differing levels of skills in the labor market and so on.

In our model the total land reserve L is utilized and no empty land is left. In practice, however, we know that in different countries there are large quantities of vacant land. The question is whether the model can be extended to allow for such a possibility and if our results may change. There are two major possibilities of extending the model to make such a result possible.

(a) Introducing the costs of transporting agricultural products which will give farmers an incentive to cluster around the cities. In this case land may be left unclaimed at greater distances from the cities. The agricultural land rent in such a case will not be constant but will gradually decline from the border of the city and reach zero value when it encounters empty land. Agricultural land rents at the city limits will be different for the two different types of cities depicted in our economy, i.e. higher in the bigger city.

(b) Allowing farmers to consume local public goods in the center of the city. This again would give farmers an incentive to cluster in proximity to the cities with the same results as in the previous case.

8. Conclusions

The main results of this paper are as follows.

(1) A local government can achieve efficiency in its jurisdiction if, except for corrective Pigouvian taxes and subsidies, the only source of financing its activities (such as the provision of desirable local public goods) is from taxes on land rents.

(2) Any tax, subsidy, and expenditure on local public goods by a local government is passed on to land rents. The welfare gains or losses of such an act are measured by the change in the total city surplus and are also passed on to land rents.

(3) A local government has an *a posteriori* criterion with which to judge the efficiency of its actions. If total land rents increase after a government action more than the expenditure on it, then this action has increased the efficiency in the city and is thus a positive step. If total land rents increased less than net expenditure on the project, then the project has contributed to a decrease in efficiency.

(4) A local government can provide its own financing to achieve local efficiency without any intervention by a federal government. An economy with local public goods can operate efficiently with only local authorities and without any central government intervention.

(5) Optimal jurisdiction of cities are those which allow acts of consumption and production of households to be performed in the same jurisdiction.

Appendix

The efficient solution of the economy is the solution to the following problem.

(a) The choice variables are: U_0, A_i, L_i, N_i, q_i, X_i, N_A, l_A, X_A, h_A, Z_i^A, Z_A^A, $h_i(D)$, $Z_j^i(D)$, $Z_A^i(D)$, $e_i(D)$, m_i (if in the long run). For definition of variables see section 2.

(b) Exogenously given parameters are: P_1, P_2, Y_0, L, N (m_i if in the short run).

(c) The problem:

$$\text{Max}\, U_0$$

subject to the following constraints.

(A1) Equal utility constraint:

$$U(h_i(D), C(q_i, D, e(D), N_i), Z_1^i(D), Z_2^i(D), Z_A^i(D)) - U_0 = 0,$$
$$U(h_A, 0, Z_1^A, Z_2^A, Z_3^A) - U_0 = 0.$$

(A2) Production technology in city i:

$$X_i = g^i(X_i) F^F(q_i, A_i, N_i).$$

(A3) Agricultural production:

$$X_A = N_A f^A(I_A).$$

(A4) Total land constraint:

$$\sum_i m_i L_i + N_A(I_A + h_A) \leqq L.$$

(A5) Total population constraint:

$$\sum m_i N_i + N_A = N.$$

(A6) International trade balance constraint:

$$\sum_{i=1}^{2} \left[P_i X_i - \int_{A_i}^{L_i} \frac{\mathrm{d}D}{h_i(D)} \left(\sum_{j=1}^{2} P_j Z_j^i(D) + e_i(D) + t(D) \right) \right]$$
$$-N_A \left(\sum_{j=1}^{2} P_j Z_j^A \right) - Y_0 = 0.$$

Note that e and t are measured in income units exchangeable with traded goods.

(A7) Balance between supply and demand of the agricultural good:

$$X_A - \sum_i m_i \left(\int_{A_i}^{L_i} \frac{Z_A^i(D)}{h_i(D)} \mathrm{d}D \right) - N_A z_A^A = 0.$$

(A8) Balance of supply and demand for labor in city i:

$$N_i - \int_{A_i}^{L_i} \frac{\mathrm{d}D}{h_i(D)} = 0.$$

(d) In the short run the number of cities, m_i, is given exogenously. In the long run m_i are decision variables. Thus, all the necessary conditions in the short run are also necessary conditions in the long run with two extra equations added.

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